NOTE

Development of the Efficiency of Yield Energy Dissipation from the Yield Strength Power Law Relationship

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INTRODUCTION

In recent years the extended use of finite element analysis, with polymeric compounds[1] and composites,[2] has generated a need for a simple analysis approach that relates creep, stress relaxation, and constant strain rate measurements all in one simple model. One such unifying model has recently been published by this author[3] that introduces a new mathematical model to describe a definitive relationship between constant strain rate, creep, and stress relaxation analysis for viscoelastic polymeric compounds. Prior to the introduction of this new model, several authors had attempted to describe two or more of these viscoelastic concepts in one unifying formulation.[4,5] However, most of the efforts over the years have been to simulate uniaxial creep,[6,7] stress relaxation,[4] or constant strain rate data[8–11] separately. This new formulation approach also offers a reasonably simple process in which to shift from a constant strain rate configuration to a creep calculation or stress relaxation configuration without changing formulation considerations or without stress or strain discontinuities.

Several well established mathematical concepts have been combined and utilized to form this new model that successfully characterizes these three viscoelastic properties. One of these mathematical concepts is the well-known power law relationship between tensile strength and time. While this simple mathematical relationship has been found to have great practical utility, it has not yet been successfully addressed from a fundamental point of view. However, this relationship between stress and time has been successfully applied to the stress relaxation of the yield point using the following simple relationship currently included in ASTM D2837–98a (Standard Test Method for Obtaining Hydrostatic Design Basis for Thermoplastic Pipe Materials):

$$\sigma_y = \frac{\beta}{t_y^n} \tag{1}$$

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where σ_y = engineering yield stress, t_y = time to yield, and β and n = constants.

This relationship has also been used by Reinhart[12] to predict long-term failure stress (which is normally close to the stress evaluated from the stress relaxation of the yield stress) as a function of time. An example of this relationship is shown in Figure 1 where n = 0.21 and β = 4900 psi. The time to reach each strain, t, (for each constant strain rate curve) in Figure 1 was calculated using the following:

$$t = \frac{\varepsilon}{\dot{\varepsilon}_i} \tag{2}$$

where ε = strain at any point in a constant strain rate curve and $\dot{\varepsilon}_i$ = constant strain rate.

Also included in Figure 1 are the stress versus time curves at several different constant strain rates; for an example, ABS material using a model published earlier by this author[3,13] with the following additional constants (ε_∞ = 0.04, ε_o = 0.0044, γ = 50 min, and K = 58). The constants for this model can be defined as: K = ratio of modulus to the yield strength, ε_∞ = long term limiting strain to yield, ε_o = supplemental strain to yield limit, and γ = exponential strain rate constant for yield strain.

It will be shown that a new analysis approach can easily be developed to address a more fundamental evaluation of the power law relationship between yield strain and the time to yield.

ANALYSIS OF THE SIMPLE POWER LAW RELATIONSHIP BETWEEN THE YIELD STRESS AND THE TIME TO YIELD

The analysis of eq. (1) can be addressed initially by simply taking the derivative of this equation to yield:

$$\frac{d\sigma_y}{dt_y} = -n\left(\frac{\beta}{t_y^{n+1}}\right) \tag{3}$$

$$\frac{d\sigma_y}{dt_y} = -n\left(\frac{\sigma_y}{t_y}\right) \tag{4}$$

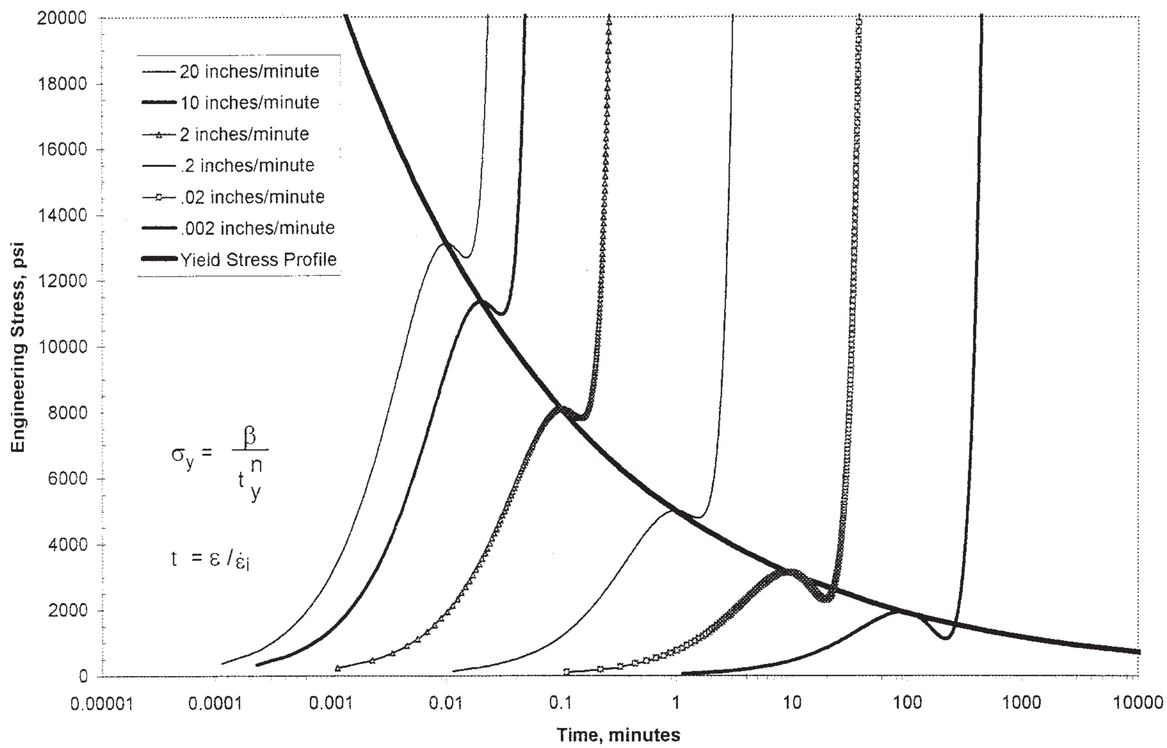

Figure 1 Calculated engineering stress vs. time at various strain rates for a simulated ABS material with indications of the engineering yield stress profile as a function of time to yield.

This equation can be further modified by noting that the time to yield, t_y, can be described in additional detail using a new viscoelastic concept model discussed elsewhere.[3,13–15] The most important consideration here is that for most materials the strain to yield, ε_y, is very nearly a constant over the full range of strain rates. For most materials the strain to yield does appear to vary only very slightly between two well defined limits over the full range of strain rates.[13] With this assumption, then, the yield strain, ε_y, and the time to yield, t_y, can be related by a characteristic strain rate, as:

$$t_y = \frac{\varepsilon_y}{\dot{\varepsilon}_i} \tag{5}$$

Combining eqs. (4) and (5) gives

$$\frac{d\sigma_y}{dt_y} = -n\left(\frac{\sigma_y}{\varepsilon_y}\right)\dot{\varepsilon}_i \tag{6}$$

or

$$\frac{d\sigma_y}{dt_y} = -n\left(\frac{\sigma_y}{\varepsilon_y}\right)\frac{d\varepsilon}{dt} \tag{7}$$

We can now define the secant yield modulus, E_y, as

$$E_y = \left(\frac{\sigma_y}{\varepsilon_y}\right) \tag{8}$$

The secant yield modulus, E_y, can be visualized as the slope of the line from the origin to the yield point. Eq. 7 can then be rewritten as

$$\frac{d\sigma_y}{dt_y} = -nE_y\frac{d\varepsilon}{dt} \tag{9}$$

At this point, notice that the dimensions of the variables on the right hand side of equation 9 give

$$-nE_y\frac{d\varepsilon}{d_t}[=]\left(\frac{\text{Force}}{(\text{Length})^2}\right)\left(\frac{\Delta\text{Length}}{(\text{Length})(\Delta\text{Time})}\right) \tag{10}$$

Therefore, the dimensions of the variables on the right hand side of equation 9 are in terms of the rate of change in energy/volume relative to time. Also note that the constant n can be considered to be a dampening factor for the rate of dissipation of the available energy/volume relative to time in going from one strain rate curve to another. This result is more clearly visualized in Figure 1. Therefore, for a specific viscoelastic material, the constant n can be defined as the *efficiency of yield energy dissipation*. For most material applications it has been found that it is very desirable to have a material with a very low *efficiency of yield energy dissipation* as indicated elsewhere in the literature.[13–15]

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